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TWO NEW OPERATORS DEFINED ON NON-NEGATIVE TRAPEZOIDAL FUZZY NUMBER MATRICES

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ABSTRACT: Fuzzy matrix is a very important topic of fuzzy algebra. Fuzzy matrices are successfully used fuzzy uncertainly occurs in a problem. The arithmetic operations of fuzzy numbers in a very important issue in fuzzy set theory, decision process, data analysis and applications. In this paper, we present two new binary fuzzy operators $\oplus$ and $\odot$ are introduced for non negative trapezoidal fuzzy number matrix. Several properties on $\oplus$ and $\odot$ are defined and satisfied. Then Numerical examples also presented.

KEYWORDS: Fuzzy number, Trapezoidal fuzzy number (TrFN), Trapezoidal fuzzy matrix (TrFM), Non- Negative Trapezoidal fuzzy matrix(Non-Negative TrFM).

I. INTRODUCTION

Fuzzy sets have been introduced by Lofti.A.Zadeh[14] Fuzzy set theory permits the gradual assessments of the membership of elements in a set which is described in the interval [0,1]. It can be used in a wide range of domains where information is incomplete and imprecise. A fuzzy number is a generalization of regular, real number in the sense that it does not refer to one single value but rather to a connected set of possible values, where each possible value has its own weight between 0 and 1. This weight is called the membership function. Interval arithmetic was first suggested by Dwyer [3] in 1951, by means of Zadeh's extension principle [15], the usual Arithmetic operations on real numbers can be extended to the ones defined on Fuzzy numbers. Dubosis and Prade [2] has defined any of the fuzzy numbers. In 1985, Chen formulated the arithmetic operations between generalized fuzzy numbers by proposing the function principle [1]. Fuzzy numbers positive and negative are given in [4].

We introduce trapezoidal fuzzy matrices (TrFMs). To the best of our knowledge, no work is available on TrFMs, through a lot of work on fuzzy matrices is available in literature. A brief review on fuzzy matrices is given below. A presented new ranking function and arithmetic operations on Some properties of constant, Adjoint and trace of trapezoidal fuzzy matrices by N.Mohana and R.Mani [6, 7, 8].

The fuzzy number can be classified in differenty forms such as triangular fuzzy number, trapezoidal fuzzy number (TrFN), L-R type fuzzy number, etc., fuzzy matrices were introduced for the first time by Thomason [13] who discussed the convergence of power of fuzzy matrix. Fuzzy matrices play an important role in scientific development. Two new operations and some applications of fuzzy matrices are given in [10, 11, 12].

Ragab et.al [9] presented some properties of the min-max composition of fuzzy matrices. Kim [5] presented some important results on determinant of square fuzzy matrices.

Firstly in section 2, we recall the definition of Trapezoidal fuzzy number and some operations on trapezoidal fuzzy numbers (TrFNs). In section 3, we have reviewed the definition of trapezoidal fuzzy matrix (TrFM) and two new operators $\oplus$ and $\odot$ on Trapezoidal fuzzy number matrices (TrFNMs). In section 4, based on some special types of trapezoidal fuzzy number matrices (TrFMs). In section 5, discusses some properties of non negative trapezoidal fuzzy number matrices over these new operators $\oplus$ and $\odot$ are presented. Finally in section 6, conclusion is included.

II. PRELIMINARIES

In this section, we recapitulate some underlying definitions and basic results of fuzzy numbers.

Definition 2.1: Fuzzy set

A fuzzy set is characterized by a membership function mapping the element of a domain, space or universe of discourse X to the unit interval [0,1]. A fuzzy set A in a universe of discourse X is defined as the following set of pairs

$$A=\{(x,\mu_A(x)) \,;\, x\in X\}$$

Here $\mu_A : X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x\in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

Definition 2.2: Fuzzy number

A fuzzy set $\tilde{A}$ defined on the set of real number R is said to be fuzzy number if its membership function has the following characteristics

i. $\tilde{A}$ is normal
ii. $\tilde{A}$ is convex
iii. The support of $\tilde{A}$ is closed and bounded then $\tilde{A}$ is called fuzzy number.

Definition 2.3: Trapezoidal fuzzy number

A fuzzy number $\tilde{A}^{TzL} = (a_1, a_2, a_3, a_4)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}^{TzL}}(x) = \begin{cases} 0 & ; \; x \le a_1 \\ \dfrac{x-a_1}{a_2-a_1} & ; \; a_1 \le x \le a_2 \\ 1 & ; \; a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3} & ; \; a_3 \le x \le a_4 \\ 0 & ; \; x > a_4 \end{cases}$$

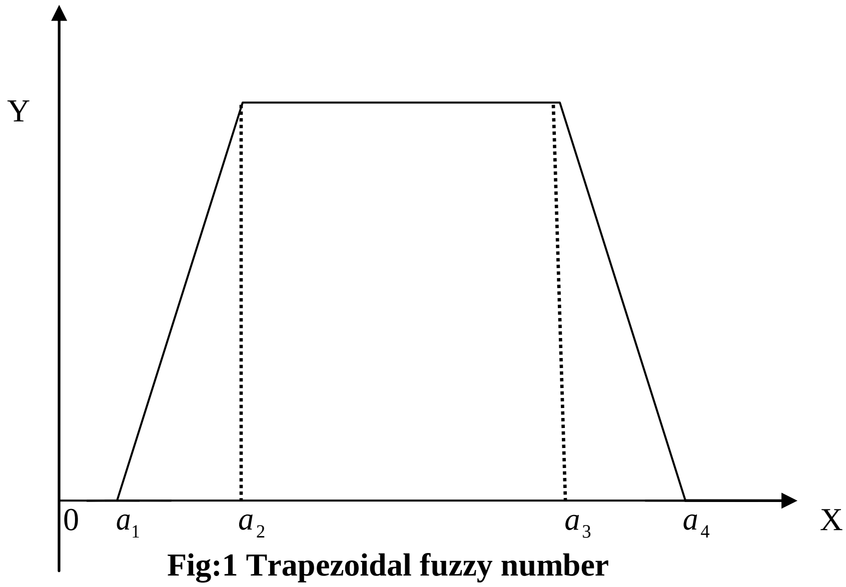

Fig:1 Trapezoidal fuzzy number

Example 2.3.1:

$\tilde{A}^{TzL} = (-2,2,4,8)$

Definition 2.4: Positive trapezoidal fuzzy number

A positive trapezoidal fuzzy number $\tilde{A}^{TzL}$ is denoted as $\tilde{A}^{TzL} = (a_1, a_2, a_3, a_4)$ where all a_i's> 0 for all $i = 1,2,3,4$.

Example 2.4.1:

$\tilde{A}^{TzL} = (1,3,4,8)$ is a positive trapezoidal fuzzy number.

Definition 2.5: Negative trapezoidal fuzzy number

A negative trapezoidal fuzzy number $\tilde{A}^{TzL}$ is denoted as $\tilde{A}^{TzL} = (a_1, a_2, a_3, a_4)$ where all a_i's< 0 for all $i = 1,2,3,4$.

Example 2.5.1:

$\tilde{A}^{TzL} = (-1, -2, -3, -6)$ is a negative trapezoidal fuzzy number.

Definition 2.6: Non-Negative trapezoidal fuzzy number

A non-negative trapezoidal fuzzy number $\tilde{A}^{TzL}$ is denoted as $\tilde{A}^{TzL} = (a_1, a_2, a_3, a_4)$ where all a_i's≥ 0 for all $i = 1,2,3,4$.

Example 2.6.1:

$\tilde{A}^{TzL} = (0,1,3,4)$ is a non-negative trapezoidal fuzzy number.

Definition 2.7: Arithmetic operations on trapezoidal fuzzy numbers (TrFNs)

Let $\tilde{A}^{TzL} = (a_1, a_2, a_3, a_4)$ and $\tilde{B}^{TzL} = (b_1, b_2, b_3, b_4)$ be trapezoidal fuzzy numbers (TrFNs) then we defined,

Addition

$\widetilde{A}^{TzL} + \widetilde{B}^{TzL} = (a_1+b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

Subtraction

$\widetilde{A}^{TzL} - \widetilde{B}^{TzL} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

Multiplication

$\widetilde{A}^{TzL} \times \widetilde{B}^{TzL} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$

III. TWO NEW OPERATIONS ⊕ AND ⊙ ON TRAPEZOIDAL FUZZY NUMBER MATRICES AND SOME OPERATIONS ON TRAPEZOIDAL FUZZY NUMBER MATRICES

In this section, we introduced the two new operations ⊕ and ⊙ on trapezoidal fuzzy matrices and some operations on trapezoidal fuzzy number matrices.

Definition 3.1: Trapezoidal fuzzy matrix (TrFM)

A trapezoidal fuzzy matrix of order m×n is defined as A= $(\tilde{a}_{ij}^{TzL})_{m\times n}$, where a_{ij}= $(a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4})$ is the ij^{th} element of A.

Definition 3.2: Some operations on Trapezoidal Fuzzy Matrices (TrFMs)

Let A = $(\tilde{a}_{ij}^{TzL})$ and B =$(\tilde{b}_{ij}^{TzL})$ be two trapezoidal fuzzy matrices (TrFMs) of same order. Then, we have the following

(i) $A \oplus B = \left[\left(\tilde{a}_{ij}^{TzL} + \tilde{b}_{ij}^{TzL}\right) - \left(\tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL}\right)\right]$

(ii) $A \odot B = \left[\tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL}\right]$

(iii) $A' = \left[\tilde{a}_{ij}^{TzL}\right]$

(iv) $A \leq B$ if and only if $\tilde{a}_{ij}^{TzL} \leq \tilde{b}_{ij}^{TzL}$ for all i, j.

IV. SOME SPECIAL TYPES OF TRAPEZOIDAL FUZZY NUMBER MATRICES (TRFM)

Let $R = \left(\tilde{r}_{ij}^{TzL}\right)$ be an $n \times n$ trapezoidal fuzzy number matrix. Then,

(i) R is reflexive if and only if $\tilde{r}_{ii}^{TzL} = 1$ for all $i = 1,2, \ldots, n$

(ii) R is irreflexive if and only if $\tilde{r}_{ii}^{TzL} = 0$ for all $i = 1,2, \ldots, n$

(iii) R is nearly irreflexive if and only if $\tilde{r}_{ii}^{TzL} \leq \tilde{r}_{ij}^{TzL}$ for all $i = 1,2, \ldots, n$.

(iv) R is symmetric if and only if $R' = R$.

(v) R is constant if and only if $\tilde{r}_{ii}^{TzL} = \tilde{r}_{jk}^{TzL}$ for all $i, j, k = 1,2, \ldots, n$.

(vi) R is identity if and only if $\tilde{r}_{ii}^{TzL} = 1$ and $\tilde{r}_{ij}^{TzL} = 0$ $(i \neq j)$ for all i, j.

The identity matrix of order $n \times n$ is generally denoted by I_n

(vii) R is weakly reflexive if $\tilde{r}_{ii}^{TzL} \geq \tilde{r}_{ij}^{TzL}$ for all i, j.

(viii) R is diagonal if $\tilde{r}_{ii}^{TzL} \geq 0, \tilde{r}_{ij}^{TzL}$ for all i, j.

V. SOME PROPERTIES ON TWO NEW OPERATORS ⊕ AND ⊙ AND SPECIAL TYPES OF NON NEGATIVE TRAPEZOIDAL

FUZZY NUMBER MATRICES

In this section, we introduced the properties on two new operators $\oplus$ and $\odot$ and special types of non negative TrFMs.

PROPOSITION 5.1:

Let A and B be two non negative trapezoidal fuzzy number matrices, then

(i) $A \oplus B \leq A \odot B$

(ii) If A and B are symmetric, then $A \oplus B$ and $A \odot B$ are symmetric.

(iii) If A and B are nearly irreflexive, then $A \odot B$ is nearly irreflexive.

PROOF:

(i) The ij^{th} element of $A \oplus B$ is $\left[\left(\tilde{a}_{ij}^{TzL} + \tilde{b}_{ij}^{TzL}\right) - \left(\tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL}\right)\right]$ and $A \odot B$ is $\left[\tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL}\right]$

$$\tilde{a}_{ij}^{TzL}\left[(1,1,1,1) - \tilde{b}_{ij}^{TzL}\right] + \tilde{b}_{ij}^{TzL}\left[(1,1,1,1) - \tilde{a}_{ij}^{TzL}\right] \leq 0 \text{ and } \tilde{a}_{ij}^{TzL}.\tilde{b}_{ij}^{TzL} \geq 0.$$

Hence $A \oplus B \leq A \odot B$.

(ii) Let $A = \left(\tilde{a}_{ij}^{TzL}\right)$ and $B = \left(\tilde{b}_{ij}^{TzL}\right)$ be two symmetric trapezoidal fuzzy number matrices. Therefore $\tilde{a}_{ij}^{TzL} = \tilde{a}_{ji}^{TzL}$ and $\tilde{b}_{ij}^{TzL} = \tilde{b}_{ji}^{TzL}$. Let $\tilde{c}_{ij}^{TzL}$ be the ij^{th} element of $A \oplus B$.

$$\tilde{c}_{ij}^{TzL} = \left(\tilde{a}_{ij}^{TzL} + \tilde{b}_{ij}^{TzL}\right) - \left(\tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL}\right) = \left(\tilde{a}_{ji}^{TzL} + \tilde{b}_{ji}^{TzL}\right) - \left(\tilde{a}_{ji}^{TzL} \,.\, \tilde{b}_{ji}^{TzL}\right) = \tilde{c}_{ji}^{TzL}.$$

Hence $A \oplus B$ is symmetric.

Let $\tilde{d}_{ij}^{TzL}$ be the ij^{th} element of $A \odot B$, $\tilde{d}_{ij}^{TzL} = \tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL} = \tilde{a}_{ji}^{TzL} \,.\, \tilde{b}_{ji}^{TzL} = \tilde{d}_{ji}^{TzL}$

Hence $A \odot B$ is symmetric.

(iii) Since A and B are nearly irreflexive $\tilde{a}_{ii}^{TzL} \leq \tilde{a}_{ij}^{TzL}$ and $\tilde{b}_{ii}^{TzL} \leq \tilde{b}_{ij}^{TzL}$ for all i, j

Let $\tilde{d}_{ij}^{TzL}$ be the ij^{th} element of $A \odot B$,

$$\tilde{d}_{ij}^{TzL} - \tilde{d}_{ii}^{TzL} = \tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL} \geq \tilde{a}_{ii}^{TzL} \,.\, \tilde{b}_{ii}^{TzL} \geq 0 \Rightarrow \tilde{d}_{ii}^{TzL} \leq \tilde{d}_{ij}^{TzL}.$$

Hence $A \odot B$ nearly irreflexive.

Example 5.1.1:

$A \oplus B \leq A \odot B$

Solution:

$$\text{Let } A = \begin{bmatrix} (0,1,3,4) & (1,3,4,8) & (1,2,3,6) \\ (1,3,4,8) & (1,2,3,6) & (2,4,6,8) \\ (0,3,4,5) & (2,4,6,8) & (1,4,5,6) \end{bmatrix} \text{and}$$

$$B = \begin{bmatrix} (0,1,3,4) & (1,3,4,8) & (0,3,4,5) \\ (1,3,4,8) & (1,2,3,6) & (2,4,6,8) \\ (1,4,5,6) & (0,2,4,6) & (2,4,8,10) \end{bmatrix}$$

LHS:

$$A \oplus B = \left[\left(\tilde{a}_{ij}^{TzL} + \tilde{b}_{ij}^{TzL}\right) - \left(\tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL}\right)\right]$$

$$\left(\tilde{a}_{ij}^{TzL}+\tilde{b}_{ij}^{TzL}\right)=\begin{bmatrix}(0,2,6,8) & (2,6,8,16) & (1,5,7,11)\\(2,6,8,16) & (2,4,6,12) & (4,8,12,16)\\(1,7,9,11) & (2,6,10,14) & (3,8,13,16)\end{bmatrix}\longrightarrow(1)$$

$$\left(\tilde{a}_{ij}^{TzL}\,.\,\tilde{b}_{ij}^{TzL}\right)=\begin{bmatrix}(0,1,9,16) & (1,9,16,64) & (0,6,12,30)\\(1,9,16,64) & (1,4,9,36) & (4,16,36,64)\\(0,12,20,30) & (0,8,24,48) & (2,16,40,60)\end{bmatrix}\longrightarrow(2)$$

$$A\oplus B=\begin{bmatrix}(-16,-7,5,8) & (-62,-10,-1,15) & (-29,-7,1,11)\\(-62,-10,-1,15) & (-34,-5,2,11) & (-60,-28,-4,12)\\(-29,-13,-3,11) & (-46,-18,2,14) & (-57,-32,-3,14)\end{bmatrix}\longrightarrow(3)$$

RHS:

$$A\odot B=\left(\tilde{a}_{ij}^{TzL}\,.\,\tilde{b}_{ij}^{TzL}\right)$$

$$\left(\tilde{a}_{ij}^{TzL}\,.\,\tilde{b}_{ij}^{TzL}\right)=\begin{bmatrix}(0,1,9,16) & (1,9,16,64) & (0,6,12,30)\\(1,9,16,64) & (1,4,9,36) & (4,16,36,64)\\(0,12,20,30) & (0,8,24,48) & (2,16,40,60)\end{bmatrix}\longrightarrow(4)$$

i.e., $\left[\left(\tilde{a}_{ij}^{TzL}+\tilde{b}_{ij}^{TzL}\right)-\left(\tilde{a}_{ij}^{TzL}\,.\,\tilde{b}_{ij}^{TzL}\right)\right]\leq\left[\left(\tilde{a}_{ij}^{TzL}\,.\,\tilde{b}_{ij}^{TzL}\right)\right]$

Hence $A\oplus B\leq A\odot B$.

Example 5.1.2:

If A and B are symmetric, then $A\oplus B$ and $A\odot B$ are symmetric.

Solution:

(i) Let $A=\begin{bmatrix}(1,1.5,2,2.5) & (1.5,2,2.5,3)\\(2,2.5,3,3.5) & (2.5,3,3.5,4)\end{bmatrix}$ and $B=\begin{bmatrix}(4,4.5,5,5.5) & (4.5,5,5.5,6)\\(5,5.5,6,6.5) & (5.5,6,6.5,7)\end{bmatrix}$

$$A\oplus B=\left[\left(\tilde{a}_{ij}^{TzL}+\tilde{b}_{ij}^{TzL}\right)-\left(\tilde{a}_{ij}^{TzL}\,.\,\tilde{b}_{ij}^{TzL}\right)\right]$$

$$\tilde{c}_{ij}^{TzL}=\left(\tilde{a}_{ij}^{TzL}+\tilde{b}_{ij}^{TzL}\right)-\left(\tilde{a}_{ij}^{TzL}\,.\,\tilde{b}_{ij}^{TzL}\right)=\left(\tilde{a}_{ji}^{TzL}+\tilde{b}_{ji}^{TzL}\right)-\left(\tilde{a}_{ji}^{TzL}\,.\,\tilde{b}_{ji}^{TzL}\right)=\tilde{c}_{ji}^{TzL}\longrightarrow(5)$$

$$\tilde{c}_{ij}^{TzL}=\begin{bmatrix}(-8.75,-4,0.25,4) & (-12,-6.75,-2,2.25)\\(-15.75,-10,-4.75,0) & (-20,-13.75,-8,-2.75)\end{bmatrix}\longrightarrow(6)$$

and $$\tilde{c}_{ji}^{TzL}=\begin{bmatrix}(-8.75,-4,0.25,4) & (-15.75,-10,-4.75,0)\\(-12,-6.75,-2,2.25) & (-20,-13.75,-8,-2.75)\end{bmatrix}\longrightarrow(7)$$

From (6) and (7) we get

Hence $A\oplus B$ are symmetric.

(ii) Let $A=\begin{bmatrix}(1,1.5,2,2.5) & (1.5,2,2.5,3)\\(2,2.5,3,3.5) & (2.5,3,3.5,4)\end{bmatrix}$ and $B=\begin{bmatrix}(4,4.5,5,5.5) & (4.5,5,5.5,6)\\(5,5.5,6,6.5) & (5.5,6,6.5,7)\end{bmatrix}$

$$A\odot B=\tilde{a}_{ij}^{TzL}\,.\,\tilde{b}_{ij}^{TzL}$$

$$\tilde{d}_{ij}^{TzL}=\tilde{a}_{ij}^{TzL}\,.\,\tilde{b}_{ij}^{TzL}=\tilde{a}_{ji}^{TzL}\,.\,\tilde{b}_{ji}^{TzL}=\tilde{d}_{ji}^{TzL}\longrightarrow(8)$$

$$\tilde{d}_{ij}^{TzL}=\begin{bmatrix}(4,6.75,10,13.75) & (6.75,10,13.75,18)\\(10,13.75,18,22.75) & (13.75,18,22.75,28)\end{bmatrix}\longrightarrow(9)$$

and $$\tilde{d}_{ji}^{TzL}=\begin{bmatrix}(4,6.75,10,13.75) & (10,13.75,18,22.75)\\(6.75,10,13.75,18) & (13.75,18,22.75,28)\end{bmatrix}\longrightarrow(10)$$

From (9) and (10) we get

Hence $A\odot B$ are symmetric.

Example 5.1.3:

If A and B are nearly irreflexive, then $A\odot B$ is nearly irreflexive.

Solution:

Let $A = \begin{bmatrix} (0,2,4,6) & (1,2,3,6) & (0,1,3,4) \\ (1,2,3,6) & (2,4,6,8) & (0,2,4,6) \\ (0,1,3,4) & (3,4,5,8) & (1,2,4,5) \end{bmatrix}$ and

$B = \begin{bmatrix} (1,2,4,5) & (2,4,6,8) & (0,2,4,6) \\ (0,1,3,4) & (0,2,4,6) & (3,4,5,8) \\ (2,3,5,6) & (3,4,6,7) & (3,4,7,10) \end{bmatrix}$ are nearly irreflexive then

$A \odot B = \begin{bmatrix} (0,4,16,30) & (2,8,18,48) & (0,2,12,24) \\ (0,2,9,24) & (0,8,24,48) & (0,8,20,48) \\ (0,3,15,24) & (9,16,30,56) & (3,8,28,50) \end{bmatrix}$ is nearly irreflexive.

PROPOSITION 5.2:

For any non negative trapezoidal fuzzy number matrix $\boldsymbol{A}, \boldsymbol{A} \odot \boldsymbol{A} \geq \boldsymbol{A}$.

PROOF:

The ij^{th} element of $\tilde{a}_{ij}^{TzL^2}$ of $A \odot A$ is greater than $\tilde{a}_{ij}^{TzL}$. Therefore $A \odot A \geq A$.

Example 5.2.1:

Let $A = \begin{bmatrix} (1,2,3,6) & (2,3,5,6) & (0,1,2,5) \\ (3,4,5,8) & (0,1,3,4) & (2,4,6,8) \\ (2,4,6,12) & (1,4,7,8) & (3,4,7,10) \end{bmatrix}$

$A \odot A = \begin{bmatrix} (1,4,9,36) & (4,9,25,36) & (0,1,4,25) \\ (9,16,25,64) & (0,1,9,16) & (2,16,36,84) \\ (4,16,36,144) & (1,16,49,64) & (9,16,49,100) \end{bmatrix}$

Therefore $A \odot A \geq A$.

PROPOSITION 5.3:

Let $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{M}$ be any three non negative trapezoidal fuzzy number matrices, then (i) $\boldsymbol{A} \oplus \boldsymbol{B} = \boldsymbol{B} \oplus \boldsymbol{A}$ $(\boldsymbol{ii})\boldsymbol{A} \odot \boldsymbol{B} = \boldsymbol{B} \odot \boldsymbol{A}$
(iii)$(\boldsymbol{A} \odot \boldsymbol{B}) \odot \boldsymbol{M} = \boldsymbol{A} \odot (\boldsymbol{B} \odot \boldsymbol{M})$

PROOF:

(i) $A \oplus B = \left[\left(\tilde{a}_{ij}^{TzL} + \tilde{b}_{ij}^{TzL}\right) - \left(\tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL}\right)\right] = \left[\left(\tilde{b}_{ij}^{TzL} + \tilde{a}_{ij}^{TzL}\right) - \left(\tilde{b}_{ij}^{TzL} \,.\, \tilde{a}_{ij}^{TzL}\right)\right] = B \oplus A$.

(ii)$A \odot B = \tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL} = \tilde{b}_{ij}^{TzL} \,.\, \tilde{a}_{ij}^{TzL} = B \odot A$.

(iii)$(A \odot B) \odot M = \left(\tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL}\right) \tilde{m}_{ij}^{TzL} = \tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL} \,.\, \tilde{m}_{ij}^{TzL} = \tilde{a}_{ij}^{TzL} \,.\, \left(\tilde{b}_{ij}^{TzL} \,.\, \tilde{m}_{ij}^{TzL}\right) = A \odot (B \odot M)$.

The operator $\oplus$ do not obey the rule $(A \oplus B) \oplus M = A \oplus (B \oplus M)$.

Example 5.3.1:

Let $\boldsymbol{A}, \boldsymbol{B}$ be any two non negative trapezoidal fuzzy number matrices, then $\boldsymbol{A} \oplus \boldsymbol{B} = \boldsymbol{B} \oplus \boldsymbol{A}$.

Solution:

Let $A = \begin{bmatrix} (1,2,3,6) & (0,2,4,6) \\ (2,4,5,9) & (2,3,5,6) \end{bmatrix}$ and $B = \begin{bmatrix} (2,3,4,7) & (1,3,5,7) \\ (2,3,4,7) & (0,2,4,6) \end{bmatrix}$

LHS:

$A \oplus B = \left[\left(\tilde{a}_{ij}^{TzL} + \tilde{b}_{ij}^{TzL}\right) - \left(\tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL}\right)\right]$

$\left(\tilde{a}_{ij}^{TzL} + \tilde{b}_{ij}^{TzL}\right) = \begin{bmatrix} (3,5,7,13) & (1,5,9,13) \\ (4,7,9,16) & (2,5,9,12) \end{bmatrix} \longrightarrow (11)$

$\left(\tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL}\right) = \begin{bmatrix} (2,6,12,42) & (0,6,20,42) \\ (4,12,20,63) & (0,6,20,36) \end{bmatrix} \longrightarrow (12)$

$A \oplus B = \begin{bmatrix} (-39,-7,1,11) & (-41,-15,3,13) \\ (-59,-13,-3,12) & (-32,-15,3,12) \end{bmatrix} \longrightarrow (13)$

RHS:

$B \oplus A = \left[\left(\tilde{b}_{ij}^{TzL} + \tilde{a}_{ij}^{TzL}\right) - \left(\tilde{b}_{ij}^{TzL} \,.\, \tilde{a}_{ij}^{TzL}\right)\right]$

$\left(\tilde{b}_{ij}^{TzL} + \tilde{a}_{ij}^{TzL}\right) = \begin{bmatrix} (3,5,7,13) & (1,5,9,13) \\ (4,7,9,16) & (2,5,9,12) \end{bmatrix} \longrightarrow (14)$

$\left(\tilde{b}_{ij}^{TzL} \,.\, \tilde{a}_{ij}^{TzL}\right) = \begin{bmatrix} (2,6,12,42) & (0,6,20,42) \\ (4,12,20,63) & (0,6,20,36) \end{bmatrix} \longrightarrow (15)$

$B \oplus A = \begin{bmatrix} (-39,-7,1,11) & (-41,-15,3,13) \\ (-59,-13,-3,12) & (-32,-15,3,12) \end{bmatrix} \longrightarrow (16)$

From (13) and (16) we get

Hence $A \oplus B = B \oplus A$.

Example 5.3.2:

Let $\boldsymbol{A}, \boldsymbol{B}$ be any two non negative trapezoidal fuzzy number matrices, then $\boldsymbol{A} \odot \boldsymbol{B} = \boldsymbol{B} \odot \boldsymbol{A}$.

Solution:

Let $A = \begin{bmatrix} (1,2,3,6) & (0,2,4,6) \\ (2,4,5,9) & (2,3,5,6) \end{bmatrix}$ and $B = \begin{bmatrix} (2,3,4,7) & (1,3,5,7) \\ (2,3,4,7) & (0,2,4,6) \end{bmatrix}$

LHS:

$A \odot B = \left(\tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL}\right)$

$\left(\tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL}\right) = \begin{bmatrix} (2,6,12,42) & (0,6,20,42) \\ (4,12,20,63) & (0,6,20,36) \end{bmatrix} \longrightarrow (17)$

RHS:

$B \odot A = \left(\tilde{b}_{ij}^{TzL} \,.\, \tilde{a}_{ij}^{TzL}\right)$

$\left(\tilde{b}_{ij}^{TzL} \,.\, \tilde{a}_{ij}^{TzL}\right) = \begin{bmatrix} (2,6,12,42) & (0,6,20,42) \\ (4,12,20,63) & (0,6,20,36) \end{bmatrix} \longrightarrow (18)$

From (17) and (18) we get

i.e., $\left(\tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL}\right) = \left(\tilde{b}_{ij}^{TzL} \,.\, \tilde{a}_{ij}^{TzL}\right)$

Hence $A \odot B = B \odot A$.

Example 5.3.3:

Let $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{M}$ be any three non negative trapezoidal fuzzy number matrices, then $(\boldsymbol{A} \odot \boldsymbol{B}) \odot \boldsymbol{M} = \boldsymbol{A} \odot (\boldsymbol{B} \odot \boldsymbol{M})$.

Solution:

Let $A = \begin{bmatrix} (1,2,3,6) & (0,2,4,6) \\ (2,4,5,9) & (2,3,5,6) \end{bmatrix}$, $B = \begin{bmatrix} (2,3,4,7) & (1,3,5,7) \\ (2,3,4,7) & (0,2,4,6) \end{bmatrix}$ and

$M = \begin{bmatrix} (0,2,4,6) & (0,2,4,6) \\ (1,2,3,6) & (1,3,5,7) \end{bmatrix}$

LHS:

$(A \odot B) \odot M = \left(\tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL}\right) .\, \widetilde{m}_{ij}^{TzL}$

$$\left(\tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL}\right) = \begin{bmatrix} (2,6,12,42) & (0,6,20,42) \\ (4,12,20,63) & (0,6,20,36) \end{bmatrix} \longrightarrow (19)$$

$$(A \odot B) \odot M = \begin{bmatrix} (0,12,48,252) & (0,12,80,252) \\ (4,24,60,378) & (0,18,100,252) \end{bmatrix} \longrightarrow (20)$$

RHS:

$A \odot (B \odot M) = \tilde{a}_{ij}^{TzL} .\left(\tilde{b}_{ij}^{TzL} .\widetilde{m}_{ij}^{TzL}\right)$

$$\left(\tilde{b}_{ij}^{TzL} .\widetilde{m}_{ij}^{TzL}\right) = \begin{bmatrix} (0,6,16,42) & (0,6,20,42) \\ (2,6,12,42) & (0,6,20,42) \end{bmatrix} \longrightarrow (21)$$

$$A \odot (B \odot M) = \begin{bmatrix} (0,12,48,252) & (0,12,80,252) \\ (4,24,60,378) & (0,18,100,252) \end{bmatrix} \longrightarrow (22)$$

From (20) and (22) we get

Hence $(A \odot B) \odot M = A \odot (B \odot M)$.

PROPOSITION 5.4:

Let $\boldsymbol{A}, \boldsymbol{B}$ be any two non negative trapezoidal fuzzy number matrices, then

(i) $(\boldsymbol{A} \odot \boldsymbol{B})' = \boldsymbol{A}' \odot \boldsymbol{B}'$

(ii) $(\boldsymbol{A} \oplus \boldsymbol{B})' = \boldsymbol{A}' \oplus \boldsymbol{B}'$

PROOF:

(i) Let $\tilde{c}_{ij}^{TzL}$ and $\tilde{d}_{ij}^{TzL}$ be the ij^{th} element of $A \odot B$ and $A' \odot B'$ respectively.
$\tilde{e}_{ij}^{TzL} = \tilde{c}_{ji}^{TzL}$ is the ij^{th} element of $(A \odot B)'$. Then $\tilde{c}_{ij}^{TzL} = \tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL}$.
Thus $\tilde{c}_{ij}^{TzL} = \tilde{a}_{ji}^{TzL} \,.\, \tilde{b}_{ji}^{TzL}$ and $\tilde{d}_{ij}^{TzL} = \tilde{a}_{ji}^{TzL} \,.\, \tilde{b}_{ji}^{TzL}$. Therefore $\tilde{d}_{ij}^{TzL} = \tilde{e}_{ij}^{TzL}$ for all i, j.

Hence $(A \odot B)' = A' \odot B'$.

(ii) Let $\tilde{c}_{ij}^{TzL}$ and $\tilde{d}_{ij}^{TzL}$ be the ij^{th} element of $A \oplus B$ and $A' \oplus B'$ respectively.
Therefore $\tilde{e}_{ij}^{TzL} = \tilde{c}_{ji}^{TzL}$ is the ij^{th} element of $(A \oplus B)'$.
Then $\tilde{c}_{ij}^{TzL} = \left(\tilde{a}_{ij}^{TzL} + \tilde{b}_{ij}^{TzL}\right) - \left(\tilde{a}_{ij}^{TzL} \,.\, \tilde{b}_{ij}^{TzL}\right)$ and

$\tilde{d}_{ij}^{TzL} = \left(\tilde{a}_{ji}^{TzL} + \tilde{b}_{ji}^{TzL}\right) - \left(\tilde{a}_{ji}^{TzL} \,.\, \tilde{b}_{ji}^{TzL}\right)$;

$\tilde{e}_{ij}^{TzL} = \left(\tilde{a}_{ji}^{TzL} + \tilde{b}_{ji}^{TzL}\right) - \left(\tilde{a}_{ji}^{TzL} \,.\, \tilde{b}_{ji}^{TzL}\right) = \tilde{d}_{ij}^{TzL}$.

Hence $(A \oplus B)' = A' \oplus B'$.

Example 5.4.1:

Let $\boldsymbol{A}, \boldsymbol{B}$ be any two non negative trapezoidal fuzzy number matrices, then $(\boldsymbol{A} \oplus \boldsymbol{B})' = \boldsymbol{A}' \oplus \boldsymbol{B}'$.

Solution:

Let $A = \begin{bmatrix} (0,1,3,4) & (2,4,6,8) \\ (0,2,4,6) & (1,2,3,6) \end{bmatrix}$ and B= $\begin{bmatrix} (2,3,4,7) & (0,2,4,6) \\ (1,2,3,6) & (2,4,6,8) \end{bmatrix}$

LHS:

$$A \oplus B = \begin{bmatrix} (-26,-8,5,11) & (-46,-18,2,14) \\ (-35,-8,3,12) & (-45,-12,1,12) \end{bmatrix}$$

$$(A \oplus B)' = \begin{bmatrix} (-26,-8,5,11) & (-35,-8,3,12) \\ (-46,-18,2,14) & (-45,-12,1,12) \end{bmatrix} \longrightarrow (23)$$

RHS:

$$A' = \begin{bmatrix} (0,1,3,4) & (0,2,4,6) \\ (2,4,6,8) & (1,2,3,6) \end{bmatrix}$$

$$B' = \begin{bmatrix} (2,3,4,7) & (1,2,3,6) \\ (0,2,4,6) & (2,4,6,8) \end{bmatrix}$$

$$A' \oplus B' = \begin{bmatrix} (-26,-8,5,11) & (-35,-8,3,12) \\ (-46,-18,2,14) & (-45,-12,1,12) \end{bmatrix} \longrightarrow (24)$$

From (23) and (24) we get

Hence $(A \odot B)' = A' \odot B'$.

Example 5.4.2:

Let $\boldsymbol{A}, \boldsymbol{B}$ be any two non negative trapezoidal fuzzy number matrices, then $(\boldsymbol{A} \odot \boldsymbol{B})' = \boldsymbol{A}' \odot \boldsymbol{B}'$.

Solution:

Let $A = \begin{bmatrix} (0,1,3,4) & (2,4,6,8) \\ (0,2,4,6) & (1,2,3,6) \end{bmatrix}$ and B= $\begin{bmatrix} (2,3,4,7) & (0,2,4,6) \\ (1,2,3,6) & (2,4,6,8) \end{bmatrix}$

LHS:

$$A \odot B = \begin{bmatrix} (0,3,12,28) & (0,8,24,48) \\ (0,4,12,36) & (2,8,18,48) \end{bmatrix}$$

$$(A \odot B)' = \begin{bmatrix} (0,3,12,28) & (0,4,12,36) \\ (0,8,24,48) & (2,8,18,48) \end{bmatrix} \longrightarrow (25)$$

RHS:

$$A' = \begin{bmatrix} (0,1,3,4) & (0,2,4,6) \\ (2,4,6,8) & (1,2,3,6) \end{bmatrix}$$

$$B' = \begin{bmatrix} (2,3,4,7) & (1,2,3,6) \\ (0,2,4,6) & (2,4,6,8) \end{bmatrix}$$

$$A' \odot B' = \begin{bmatrix} (0,3,12,28) & (0,4,12,36) \\ (0,8,24,48) & (2,8,18,48) \end{bmatrix} \longrightarrow (26)$$

From (25) and (26) we get

Hence $(A \odot B)' = A' \odot B'$.

VI CONCLUSION

In this article, we have concentrate the notion of two new operators on Trapezoidal fuzzy number matrices are defined. Using these some relevant properties of non negative trapezoidal fuzzy number matrices are presented.

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